

Quantum Teleportation with Remote Rotation on a GHZ state

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Abstract

This study proposes a pioneering protocol for teleporting an arbitrary single particle state and simultaneously performing a rotation operation on that particle. There are protocols for either only teleporting particles or only remotely controlling quantum particles. If one has to remotely control a teleported quantum, then he/she has to first do the quantum teleportation and then perform the remote control on the teleported quantum. Both operations were done separately on two sets of entanglements. However, this intuitive solution is inefficient because many resources are wasted. Therefore, the study attempts to complete both operations using only one Greenberger-Horne-Zeilinger (GHZ) state.

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1 Introduction

The study of quantum information theory is an extended research in recent years. In the quantum information theory, the entanglement is a key physical property and has been employed in many applications (e.g., quantum teleportation [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14], quantum key distribution [15, 16, 17, 18, 19, 20], quantum secret sharing [21, 22, 23, 24, 25, 26, 27, 28, 29, 30] and etc.). Among these applications, *quantum teleportation* is an epochal application, in which an arbitrary particle can be transmitted to a distant place through the property of entanglement and some auxiliary classical communications without using any physical quantum channel. In 1993, Bennett and Brassard proposed the first teleportation protocol [7], in which two participants (called as Alice and Bob) in a distance away pre-share a two-particle entanglement quantum state, Einstein-Podolsky-Rosen (EPR) state. And then Alice can use the entanglement of EPR state to teleport an arbitrary single particle ($\alpha|0\rangle + \beta|1\rangle$) to Bob. Since Bennett and Brassard's teleportation protocol [7], many quantum teleportation protocols [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14] have been introduced to teleport various qubits via different entanglement quantum state (e.g., GHZ class state, W class state, and etc.) subsequently.

Except for teleporting an arbitrary particle, the entanglement can also be employed to transmit the information of a unitary operator. Suppose that a participant, Alice, wants to perform an arbitrary unitary operator on a particle held by a remote participant, Bob, but she is unable to immediately transmit the information of a unitary operator to Bob or she would not like

to let Bob know what the operator is. Similar to quantum teleportation, Alice and Bob can use the correlation of a pre-shared entanglement state to complete the task. This is the so called *quantum remote control*. Since the first quantum remote control protocol presented by Huelga et al. [31] in 2001, various quantum remote control protocols [32, 33, 34, 35, 36] have been introduced.

Let us consider a situation which may be useful in quantum cryptography, such as quantum secret sharing or quantum private comparison and etc.. Bob wants to teleport an arbitrary qubit $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Charlie, and Alice wants to do a unitary operator on the particle $|\varphi\rangle$. In order to achieve the task by using the existing protocols, an intuitive solution can be adopted. That is, Bob first uses the quantum teleportation with an EPR state to teleport the particle $|\varphi\rangle$ to Charlie, and then Alice employs the quantum remote control with the other EPR state to perform her operation on the particle $|\varphi\rangle$ held by Charlie. In total, 2 EPR states, 5 unitary operators and 4 classical bits were needed in this intuitive solution. This paper attempts to provide a more efficient protocol for this situation. We aim to perform the quantum teleportation and quantum remote control simultaneously by using one three-particle GHZ state.

The rest of this paper is constructed as follows. Section 2 first introduces the proposed protocol. Then, Section 3 shows a comparison of our protocol to the intuitive solution. Finally, a short conclusion is given in Section 4.

2 The Proposed Protocol

Assume there are three participants Alice, Bob and Charlie in the protocol. Suppose that Bob wants to teleport an arbitrary single particle $|\psi\rangle_b = \alpha|0\rangle_b + \beta|1\rangle_b$ to Charlie. At the same time, Alice wants to remotely perform a rotation operator $R_z(\theta)$ (about the Z axis) on $|\psi\rangle_b$, where $\theta \in [0, 2\pi]$,

$$R_z(\theta) \equiv e^{-i\theta Z/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}. \quad (1)$$

This will produce a new quantum state:

$$|\varphi\rangle_b = R_z(\theta) |\psi\rangle_b = \alpha e^{-i\theta/2} |0\rangle_b + \beta e^{i\theta/2} |1\rangle_b. \quad (2)$$

In order to complete the above tasks, let Alice, Bob and Charlie pre-share a three-particle GHZ state

$$|\phi\rangle_{q_1 q_2 q_3} = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{q_1 q_2 q_3}, \quad (3)$$

where Alice, Bob and Charlie have the first, the second and the third particles, respectively. The proposed protocol is described in the following steps:

Step1 Bob first performs a joint measurement on his particle pair (b, q_2) with the Bell basis $\{|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}, |\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}\}$. The state of the composite quantum system is showed as follow

$$\begin{aligned} |\Pi\rangle_{b q_1 q_2 q_3} &= |\psi\rangle_b |\phi\rangle_{q_1 q_2 q_3} \\ &= \frac{1}{2} [|\Phi^+\rangle_{b q_2} (\alpha |00\rangle + \beta |11\rangle)_{q_1 q_3} \end{aligned}$$

$$\begin{aligned}
& + |\Phi^-\rangle_{bq_2} (\alpha |00\rangle - \beta |11\rangle)_{q_1 q_3} \\
& + |\Psi^+\rangle_{bq_2} (\alpha |11\rangle + \beta |00\rangle)_{q_1 q_3} \\
& + |\Psi^-\rangle_{bq_2} (\alpha |11\rangle - \beta |00\rangle)_{q_1 q_3}]. \tag{4}
\end{aligned}$$

After Bob performs the measurement, he broadcasts his measurement result, MR_B , to Alice and Charlie via a classical channel.

Step2 Alice performs the rotation of an angle θ on the first particle of the GHZ state, q_1 , according to MR_B . If MR_B is $|\Phi^\pm\rangle$, Alice performs $R_z(\theta)$ on q_1 . Otherwise, Alice performs $R_z(-\theta)$ on q_1 . Then, Alice measures it with X basis $\{|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}, |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}\}$ to obtain the measurement result, MR_A , which will be sent to Charlie via a classical channel.

Step3 According to MR_A and MR_B , Charlie can perform a corresponding unitary operation (shown in Table 1) on q_3 to adjust the state to the one given in Eq.(2).

Table 1: Measurement results and the corresponding operations.

MR_A	MR_B	operation
$ +\rangle$	$ \Phi^+\rangle$	I
$ -\rangle$	$ \Phi^+\rangle$	σ_z
$ +\rangle$	$ \Phi^-\rangle$	σ_z
$ -\rangle$	$ \Phi^-\rangle$	I
$ +\rangle$	$ \Psi^+\rangle$	σ_x
$ -\rangle$	$ \Psi^+\rangle$	$i\sigma_y$
$ +\rangle$	$ \Psi^-\rangle$	$i\sigma_y$
$ -\rangle$	$ \Psi^-\rangle$	σ_x

As an example, let MR_B be $|\Phi^-\rangle_{bq_2}$. The state of the composite quantum system is showed as follows:

$${}_{bq_2} \langle \Phi^- | |\Pi\rangle_{bq_1q_2q_3} = \alpha |00\rangle_{q_1q_3} - \beta |11\rangle_{q_1q_3}. \quad (5)$$

Suppose Alice's measurement result is $|+\rangle_{q_1}$. The state of q_3 held by Charlie will be $\alpha e^{-i\theta/2} |0\rangle_{q_3} - \beta e^{i\theta/2} |1\rangle_{q_3}$. Once Charlie performs a unitary operation σ_z on q_3 , he will obtain the state in Eq.(2).

3 Comparison

In the intuitive solution, Bob first has to use the quantum teleportation to teleport a particle $|\psi\rangle_b$ to Charlie which requires an EPR state, a corresponding operator and 2 bits of classical message [7]. After that, Alice employs the quantum remote control to perform her operation on the particle $|\psi\rangle_b$ held by Charlie.

In the remote control protocol, Alice and Charlie also have to pre-share a Bell state, where the subscripts q'_1 and q'_2 represent the first and the second particles of the Bell state belonged to Alice and Charlie, respectively,

$$|\Phi^+\rangle_{q'_1q'_2} = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)_{q'_1q'_2}. \quad (6)$$

Charlie first performs a Controlled-NOT operation on his qubit pair (q'_2, b) with q'_2 as the control bit and b as the target bit. Then, Charlie measures b with Z basis, and the composite state becomes

$$|\omega\rangle_{q'_1 q'_2 b} = \frac{1}{2} \left[(\alpha |00\rangle + \beta |11\rangle)_{q'_1 q'_2} |0\rangle_b + (\alpha |11\rangle + \beta |00\rangle)_{q'_1 q'_2} |1\rangle_b \right]. \quad (7)$$

Then, Charlie sends a one-bit classical message to Alice about his measurement. If the measurement result is $|0\rangle$, Charlie and Alice will do nothing. Otherwise, they both perform σ_x on their particles. So the state they shared becomes

$$|\omega'\rangle_{q'_1 q'_2} = (\alpha |00\rangle + \beta |11\rangle)_{q'_1 q'_2}. \quad (8)$$

After that, Alice performs $R_z(\theta)$ on her particle q'_1 and measures it with X basis. Alice sends a one-bit classical message to Charlie about her measurement. According to that, Charlie performs a corresponding unitary operation to complete the remote control process. Hence, the remote control requires an EPR state, 4 unitary operators and 2 bits of classical message in 2 rounds of transmission.

On the other hand, in the proposed protocol, a GHZ state is used to teleport the particle as well as remotely control the particle. In total, the proposed protocol requires a three-particle GHZ state, 2 unitary operators and 3 bits of classical message in 2 rounds of transmission. Obviously, it is more efficient than the intuitive solution (see also Table 2).

Table 2: The comparison of the proposed protocol to the intuitive solution

	Intuitive solution	Our protocol
Entanglement state	2 EPR states	1 GHZ state
Unitary operator	5	2
Number of rounds in classical transmission	3	2
Classical message	4 bits	3 bits
Measurement	1 Bell measurement 2 single-photon measurement	1 Bell measurement 1 single-photon measurement

4 Conclusion

This paper uses the entanglement in GHZ state to perform both the teleportation of an arbitrary single quantum state and the remote control of that quantum state simultaneously. The proposed protocol is more efficient than an intuitive solution, that performs quantum teleportation and quantum remote control separately in two distinct entanglements of quantum state. It should be noted that in the proposed protocol, Alice can only perform the rotation operator under Z axis, and Bob can only teleport a single particle. How to design a protocol for Alice to perform an arbitrary operator and Bob to teleport various quantum states is a promising future research.

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